

## 1.3 Cross Sections of Solids

**Essential Question** How can you use a piece of food to create a real-life cross section?

Imagine cutting through a piece of food. The intersection formed by the cut you make and the piece of food is called a **cross section**. The shape of the cross section depends on the angle of the cut you make.

### EXPLORATION 1 Describing Cross Sections

**Work with a partner.** Describe the shapes of the cross sections that are formed by making cuts into each given food so that it is separated into two congruent parts.

a. wheel of cheese



b. watermelon



c. stick of butter



d. cucumber



### EXPLORATION 2 Forming Cross Sections

**Work with a partner.** Describe how you can slice the portion of cheese so that the cross section formed is the given shape.

- a. triangle
- b. rectangle
- c. trapezoid



### ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

### Communicate Your Answer

3. How can you use a piece of food to create a real-life cross section?
4. Is there more than one way to slice the portion of cheese in Exploration 2 to form a triangular cross section? Explain. Use drawings to support your answer.

# 1.3 Lesson

## Core Vocabulary

cross section, p. 18

### Previous

plane

Pythagorean Theorem

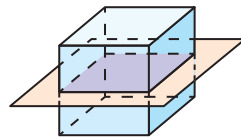
square root

## What You Will Learn

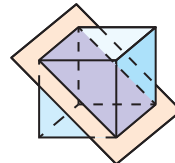
- ▶ Describe cross sections.
- ▶ Draw cross sections.
- ▶ Solve real-life problems involving cross sections.

## Describing Cross Sections

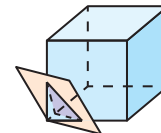
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



square



rectangle



triangle

## STUDY TIP

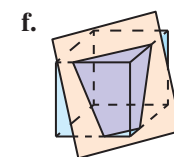
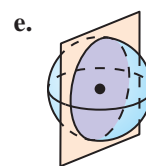
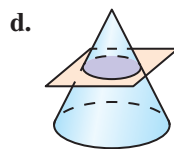
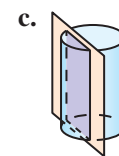
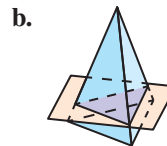
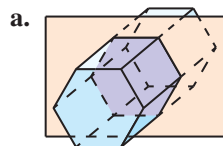
To help you better visualize a cross section, rotate the solid mentally so you are looking directly at the cross section.



### EXAMPLE 1

## Describing Cross Sections

Describe the shape formed by the intersection of the plane and the solid.



## SOLUTION

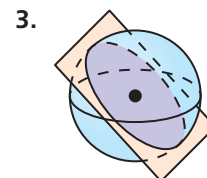
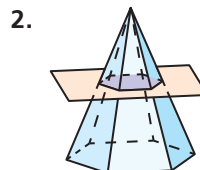
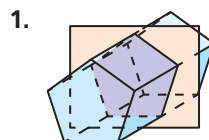
- a. The cross section is a hexagon.                      b. The cross section is a triangle.  
c. The cross section is a rectangle.                      d. The cross section is a circle.  
e. The cross section is a circle.                      f. The cross section is a trapezoid.

## Monitoring Progress



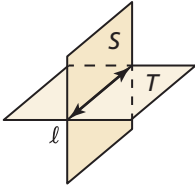
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Describe the shape formed by the intersection of the plane and the solid.



## REMEMBER

An example of the Plane Intersection Postulate is shown below, where the intersection of plane  $S$  and plane  $T$  is line  $\ell$ .

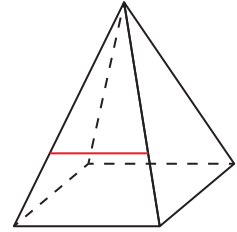


## Drawing Cross Sections

The Plane Intersection Postulate states that if two planes intersect, then their intersection is a line. This postulate can help you when drawing a cross section.

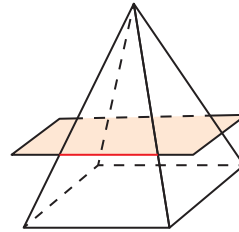
### EXAMPLE 2 Drawing a Cross Section

Draw the cross section formed by a plane parallel to the base that intersects the red line segment drawn on the square pyramid. What is the shape of the cross section?

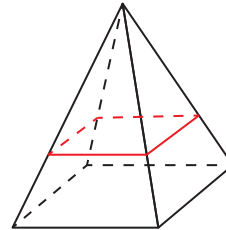


### SOLUTION

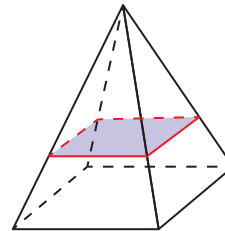
**Step 1** Visualize a horizontal plane parallel to the base that intersects the lateral face and passes through the red line segment.



**Step 2** The horizontal plane is parallel to the base of the pyramid. So, draw each pair of parallel line segments where the plane intersects the lateral faces of the pyramid.



**Step 3** Shade the cross section.



► The cross section is a square.

### Monitoring Progress



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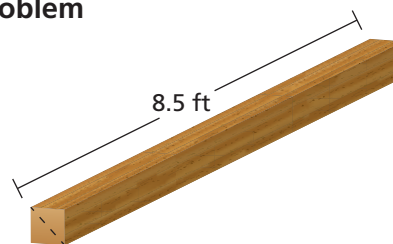
- WHAT IF?** Draw the cross section formed by a plane perpendicular to the base that intersects the vertex of the square pyramid in Example 2. What is the shape of the cross section?
- REASONING** Describe how a plane can intersect the pyramid in Example 2 so that it forms a cross section that is (a) a trapezoid and (b) a line segment.

## Solving Real-Life Problems

### EXAMPLE 3 Solving a Real-Life Problem



A machine at a sawmill cuts a 4-inch by 4-inch piece of wood lengthwise along its diagonal, as shown. Find the perimeter and area of the cross section formed by the cut.



### SOLUTION

- 1. Understand the Problem** You know that the piece of wood is shaped like a rectangular prism with a length of 8.5 feet and a width and height of 4 inches. You are asked to calculate the perimeter and area of the cross section formed when an 8.5-foot cut is made along its diagonal.
- 2. Make a Plan** Determine the shape and the dimensions of the cross section. Then use the dimensions to calculate the perimeter and area of the cross section.
- 3. Solve the Problem** Draw a diagram of the cross section. It is a rectangle with a length of 8.5 feet, or 102 inches.



Use the Pythagorean Theorem to find its width. The length and width of the end of the piece of wood is 4 inches.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 4^2 + 4^2 \quad \text{Substitute.}$$

$$c^2 = 16 + 16 \quad \text{Multiply.}$$

$$c^2 = 32 \quad \text{Add.}$$

$$c = \sqrt{32} \quad \text{Find the positive square root.}$$

$$c = 4\sqrt{2} \quad \text{Simplify.}$$

The width of the rectangular cross section is  $4\sqrt{2}$  inches.

#### Perimeter of cross section

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(102) + 2(4\sqrt{2}) \\ &= 204 + 8\sqrt{2} \\ &\approx 215.31 \end{aligned}$$

#### Area of cross section

$$\begin{aligned} A &= \ell w \\ &= 102 \cdot 4\sqrt{2} \\ &= 408\sqrt{2} \\ &\approx 577 \end{aligned}$$

▶ The perimeter of the cross section is about 215.31 inches and the area of the cross section is about 577 square inches.

- 4. Look Back** You can use estimation to check that your answer is reasonable. The length of the rectangular cross section is about 9 feet and its width is about 0.5 foot.

**Perimeter of cross section:**  $P = 2\ell + 2w = 2(9) + 2(0.5) = 19 \text{ ft} = 228 \text{ in.}$  ✓

**Area of cross section:**  $A = \ell w = 9 \cdot 0.5 = 4.5 \text{ ft}^2 = 648 \text{ in.}^2$  ✓

### Monitoring Progress



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6. A 6-inch by 6-inch piece of wood that is 10.25 feet long is cut lengthwise along its diagonal. Find the perimeter and area of the cross section formed by the cut.

# 1.3 Exercises

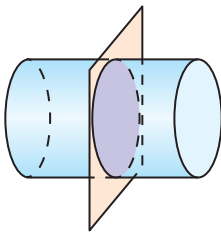
## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** The intersection of a plane and a solid is called a \_\_\_\_\_.
- WRITING** Can a plane intersect a rectangular prism and form a cross section that is a circle? Explain.

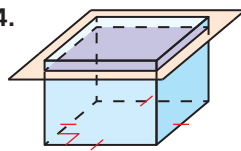
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, describe the shape formed by the intersection of the plane and the solid. (See Example 1.)

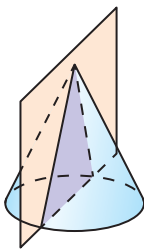
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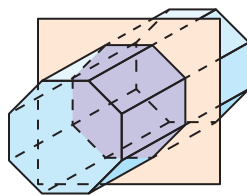
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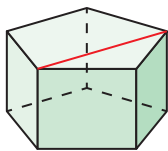


6.

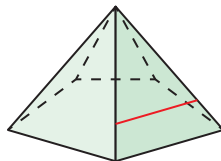


In Exercises 7–10, draw the cross section formed by the described plane that intersects the red line segment drawn on the solid. What is the shape of the cross section? (See Example 2.)

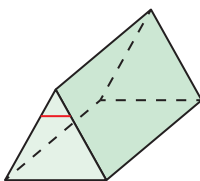
7. plane is perpendicular to base



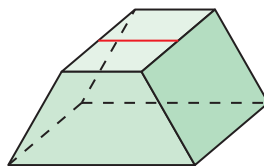
8. plane is parallel to base



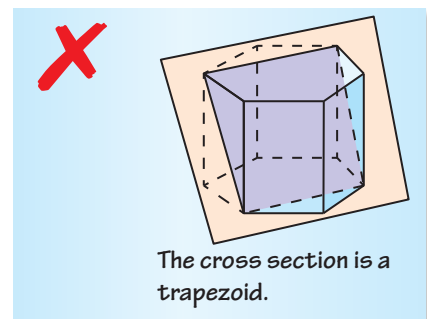
9. plane is parallel to bottom face



10. plane is perpendicular to bottom face



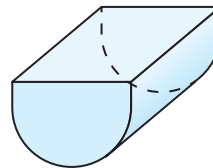
11. **ERROR ANALYSIS** Describe and correct the error in describing the shape formed by the intersection of the plane and the regular hexagonal prism.



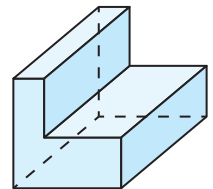
12. **OPEN-ENDED** Give an example of a solid from which a triangular, hexagonal, and trapezoidal cross section can be formed.

In Exercises 13–18, draw the cross section formed by a vertical plane that divides the solid into two congruent parts. Is there more than one way to use a vertical plane to divide the figure into two congruent parts? If so, does the cross section change? Explain.

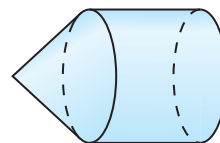
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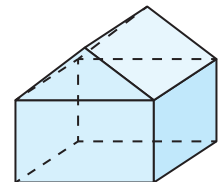
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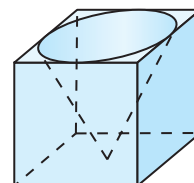
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16.



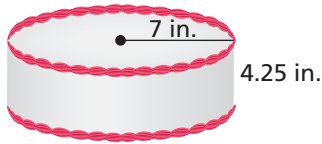
17.



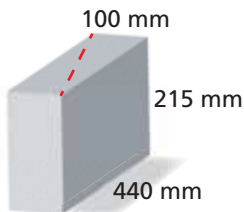
18.



19. **PROBLEM SOLVING** You cut the cake vertically to make two congruent parts. (See Example 3.)



- Find the perimeter and area of the cross section formed by the cut.
  - Find the surface area of the cake that is not frosted before the cut. How does the unfrosted surface area change after the cut?
  - Can the cake be cut another way to make two congruent parts? If so, find the perimeter and area of the cross section formed by the cut.
20. **PROBLEM SOLVING** A mason uses a concrete saw to cut the block along the indicated diagonal.



- Identify the solids formed by the cut.
  - Find the perimeter and area of the cross section formed by the cut.
  - The block has a density of about 0.002 gram per cubic millimeter. Find the mass of the block to the nearest gram.
21. **PROBLEM SOLVING** Use the figure shown.

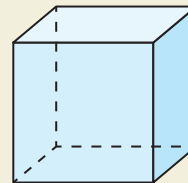


- One of the hexagonal pipes is cut vertically so that it is divided into two congruent parts. Draw two possible cross sections.
- How many different ways can a pipe be cut lengthwise to form two congruent parts? Explain.

22. **PROBLEM SOLVING** A regular octagonal pyramid is intersected by a plane perpendicular to its base. The plane passes through its vertex so it is divided into two congruent parts. Draw the cross section. Is there more than one way to divide the pyramid into two congruent parts? If so, does the shape of the cross section change? Explain.

23. **MAKING AN ARGUMENT** Your friend says that any plane that intersects a sphere forms a circular cross section. Is your friend correct? Explain.

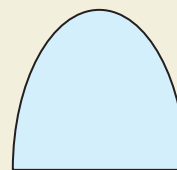
24. **HOW DO YOU SEE IT?** Draw a plane that intersects the cube to form the given cross section.



- |                       |                         |
|-----------------------|-------------------------|
| a. isosceles triangle | b. equilateral triangle |
| c. scalene triangle   | d. parallelogram        |
| e. pentagon           | f. hexagon              |

25. **REASONING** A plane intersects a sphere 7 meters from the center of the sphere. The radius of the sphere is 25 meters. Draw a diagram to represent this situation. Then find the area of the cross section to the nearest tenth.

26. **THOUGHT PROVOKING** Describe a solid that can be intersected by a plane to form the cross section shown. Explain how you form the cross section.

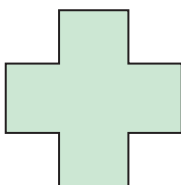


## Maintaining Mathematical Proficiency

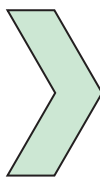
Reviewing what you learned in previous grades and lessons

Tell how many lines of symmetry the figure has. (Skills Review Handbook)

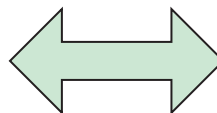
27.



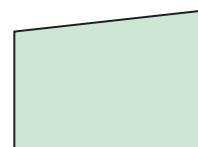
28.



29.



30.



Graph the inequality in a coordinate plane. (Skills Review Handbook)

31.  $x > 0$

32.  $x \leq \frac{1}{2}$

33.  $y \geq 0$

34.  $y < -2.5$