Unit 4B: Graphing Rational Functions

I CAN:

- Identify the important features of rational functions…
  - x- and y-intercepts
  - vertical, horizontal and/or slant asymptotes
  - holes
  - domain and range
- Graph rational functions

Monday	Tuesday	Wednesday	Thursday	Friday
22 DAY 1 Holes, Asymptotes, and Intercepts	23 DAY 2 Graphing Rational Functions	24 DAY 3 Graphing with Slant Asymptote	25 DAY 4 More Graphing Practice **DeltaMath Skills Check**	26 Help Sessions
29 DAY 5 Review	30 DAY 6 Review **Unit 4B Test**	31 Help Sessions	1 **Unit 4B Test Due 8 am**	2

THIS PLAN IS SUBJECT TO CHANGE. PLEASE REFER TO CTLS DIGITAL CLASSROOM FOR UPDATES.

Graphing Rational Functions

A rational function is a function of the form: [ ] where $p(x)$ and $q(x)$ are ____________________ and $q(x) \neq$____________

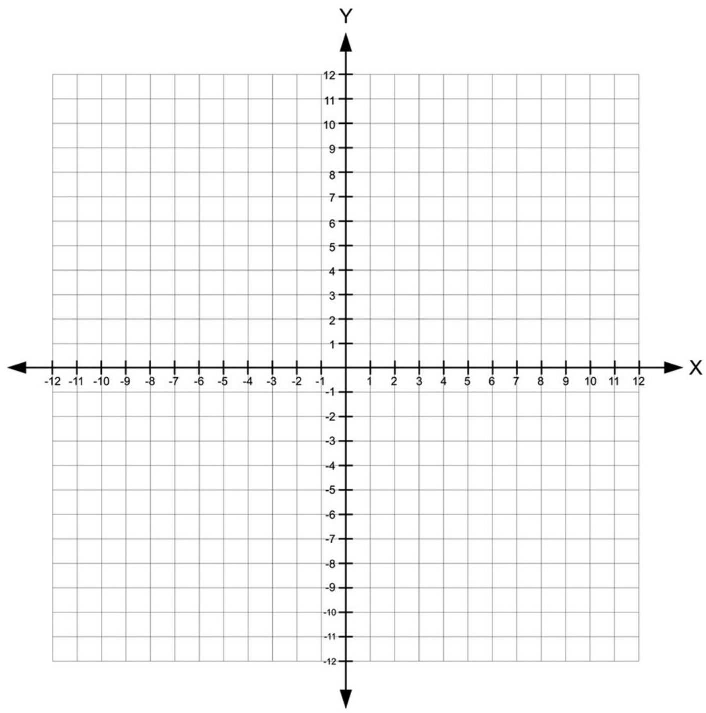
Key Features of Rational Functions	
① Simplify the function. Factor the numerator and denominator, then eliminate common factors.	
HOLES A hole is a point (x, y) at which there is a __________________ in the graph. A hole occurs when there is a ________________________________ between the numerator and denominator.	② For each factor you eliminated in step 1, there is a hole! Locate each hole: ➤ To find the x-coordinate, set the factor = 0 and solve. ➤ To find the y-coordinate, substitute the x-coordinate into the simplified function.
X-INTERCEPT(S) The points where the graph crosses the x-axis and where $y = 0$.	③ Set the **numerator = 0** and solve.
VERTICAL ASYMPTOTE(S) Vertical boundary lines which the graph will not cross, written in the form $x = a$. These occur because the denominator of a fraction cannot = ____!	④ Set the **denominator = 0** and solve.
HORIZONTAL ASYMPTOTES Horizontal guidelines, written in the form $y = a$.	⑤ Follow the rules below.

CASE	HORIZONTAL ASYMPTOTE
degree of $p <$ degree of q	
degree of $p =$ degree of q	
degree of $p >$ degree of q	

SLANT ASYMPTOTE If the degree of p is greater than the degree of q by 1, then the function has a slant asymptote in the form $y = mx + b$.	To find the slant asymptote, divide the numerator by the denominator using long or synthetic division. The **depressed polynomial** defines the asymptote – ignore the remainder.
Y-INTERCEPT The point where the graph crosses the y-axis and where $x = 0$.	Substitute 0 for x in the simplified equation and solve for y.

Ex 1: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{x - 4}{x + 4}$$



Hole(s):

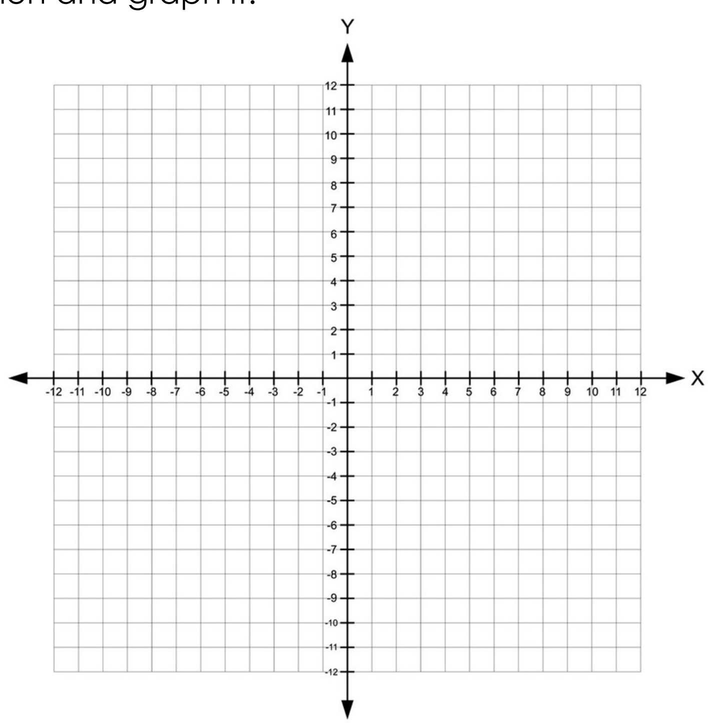
VA: x-int:

HA: y-int:

SA:

Ex 2: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{5x - 10}{x^2 - 4}$$



Hole(s):

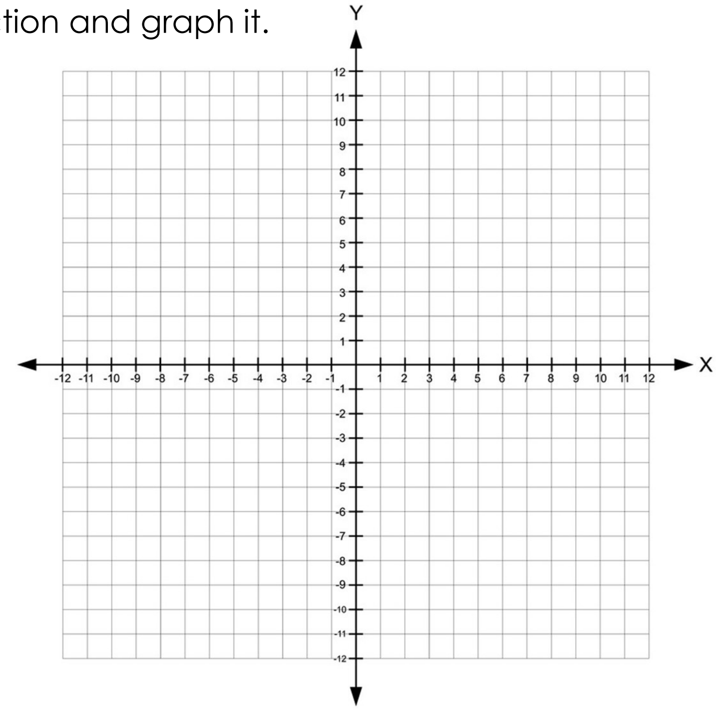
VA: x-int:

HA: y-int:

SA:

Ex 3: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{2x^2 - 8}{x^2 - x - 6}$$



Hole(s):

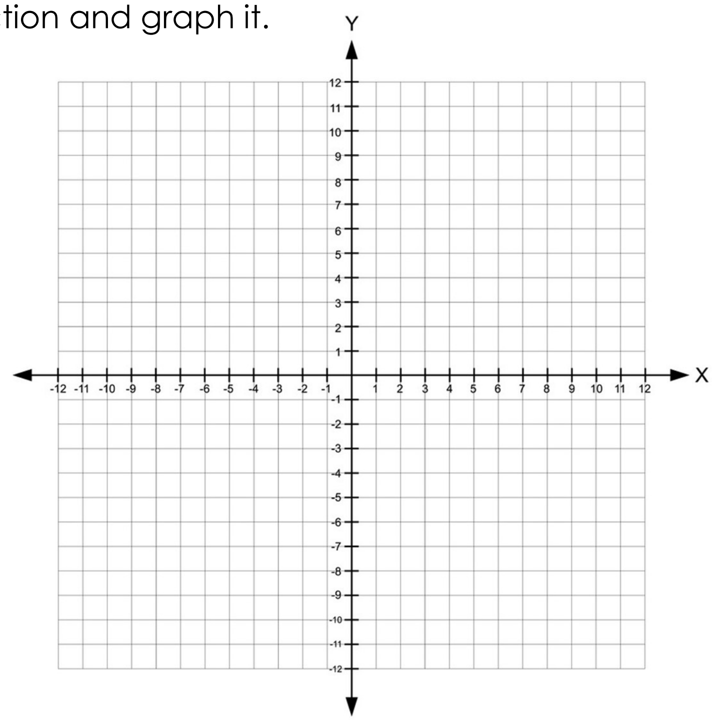
VA: x-int:

HA: y-int:

SA:

Ex 4: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{3x}{x^2 - 9}$$



Hole(s):

VA: x-int:

HA: y-int:

SA:

Check your own understanding:

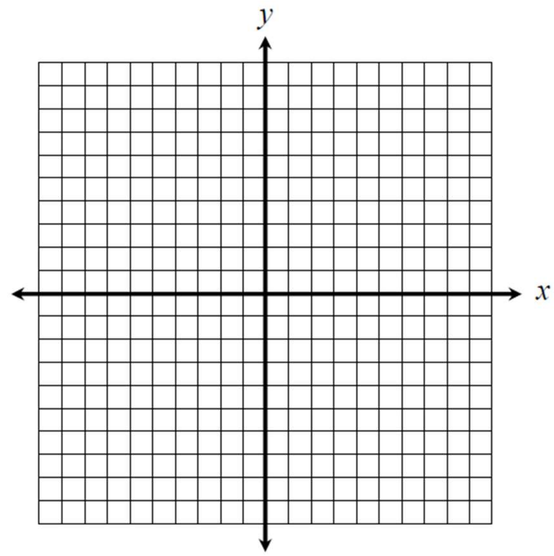
1. Circle each function that has a horizontal asymptote at $y = 0$.				
$f(x) = \frac{x+1}{x^2-9}$	$f(x) = \frac{2x+2}{x-3}$	$f(x) = \frac{x^2-4}{2x}$	$f(x) = \frac{3x}{2x^2-6x}$	$f(x) = \frac{x}{x^2-1}$
2. Circle each function that has a vertical asymptote at $x = 3$.				
$f(x) = \frac{x+1}{x^2-9}$	$f(x) = \frac{2x+2}{x-3}$	$f(x) = \frac{x^2-4}{2x}$	$f(x) = \frac{3x}{2x^2-6x}$	$f(x) = \frac{x}{x^2-1}$
3. Circle the x-intercepts of the function $f(x) = \frac{x^2-4}{2x}$? -4 -2 0 2 4				

Algebra 2 Unit 4B Name ______________________________

Graphing Rational Equations Practice

Identify the key characteristics of the function and graph it.

1. $f(x) = \frac{2x-6}{x-1}$



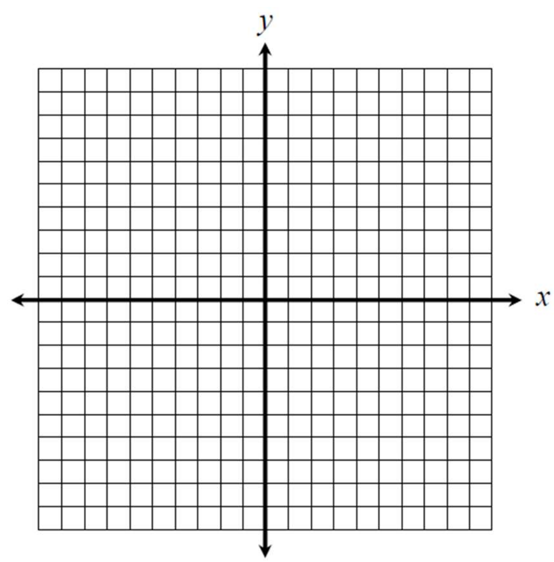
Hole(s):

VA: x-int:

HA: y-int:

SA:

2. $f(x) = \frac{x+5}{x^2+9x+20}$



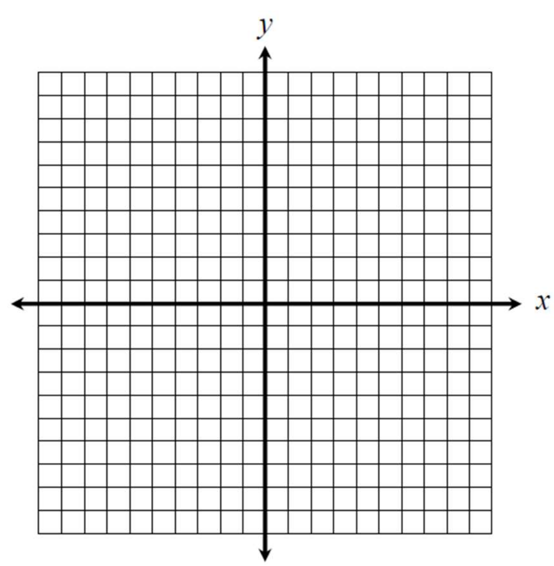
Hole(s):

VA: x-int:

HA: y-int:

SA:

3. $f(x) = \frac{-x^2+7x-12}{x^2-3x}$



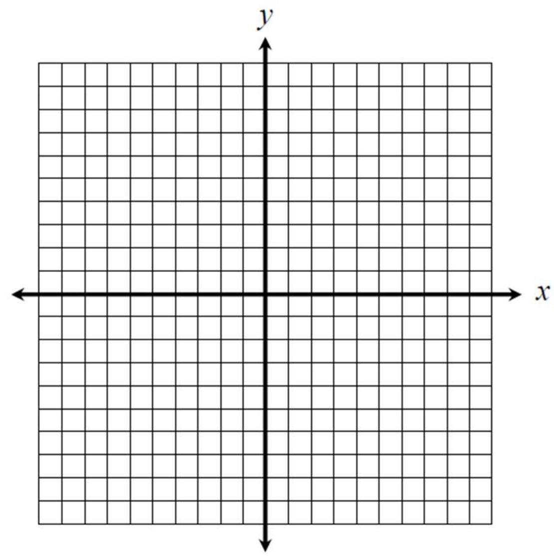
Hole(s):

VA: x-int:

HA: y-int:

SA:

4. $f(x) = \frac{x-6}{x^2-6x}$



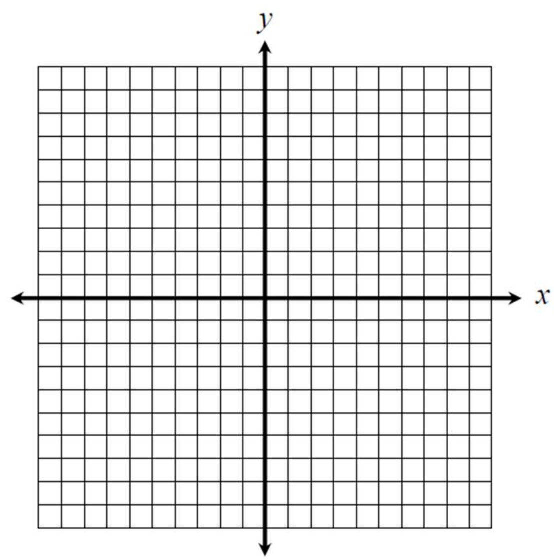
Hole(s):

VA: x-int:

HA: y-int:

SA:

5. $f(x) = \frac{x^2+3x-28}{x^2+12x+35}$



Hole(s):

VA: x-int:

HA: y-int:

SA:

Fill in the blank using words in the word bank.

DEGREES	DENOMINATOR	COEFFICIENTS	FACTOR	
ELIMINATE	MULTIPLY	NUMERATOR	X	Y

6. The first step in graphing a rational function is to _______________ the numerator and denominator and simplify the equation.

7. When the equation is simplified, factors that ________________ create holes in the graph.

8. Determine the equation of the vertical asymptote by setting the ______________________ equal to zero.

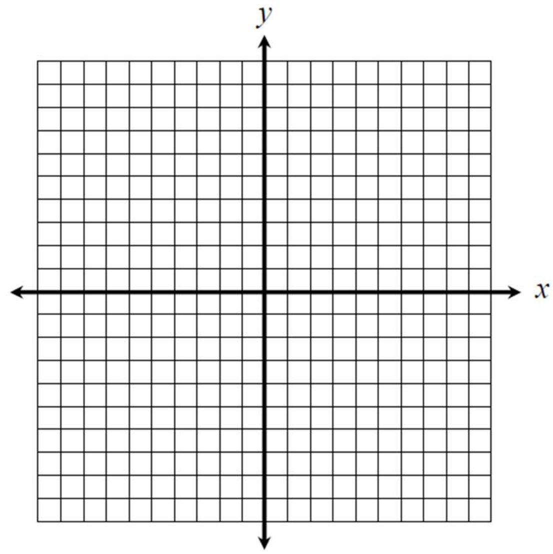
9. Determine the horizontal asymptote by comparing the _______________ of the numerator and denominator.

10. Find the x-intercepts of a rational function by setting the ______________________ equal to zero.

11. Determine the y-intercept of the function by substituting zero for _______.

More Graphing Rational Functions

Ex 1: Graph $f(x) = \frac{x^3+3x^2}{x^2+2x-3}$



Hole(s):

VA: x-int:

HA: y-int:

SA:

Finding Slant Asymptote: $y = mx + b$

Divide the numerator by the denominator. The depressed polynomial is the "$mx + b$" of the slant asymptote – ignore the remainder!

Ex 2: Determine the equations of the vertical asymptote(s) and horizontal or slant asymptotes for each function.

	VERTICAL ASYMPTOTE(S)	HORIZONTAL / SLANT
a. $f(x) = \frac{x+3}{2x}$		
b. $f(x) = \frac{x^2-5x+6}{x-1}$		
c. $f(x) = \frac{x-4}{x^2-x-6}$		
d. $f(x) = \frac{x^2-4}{x+3}$		

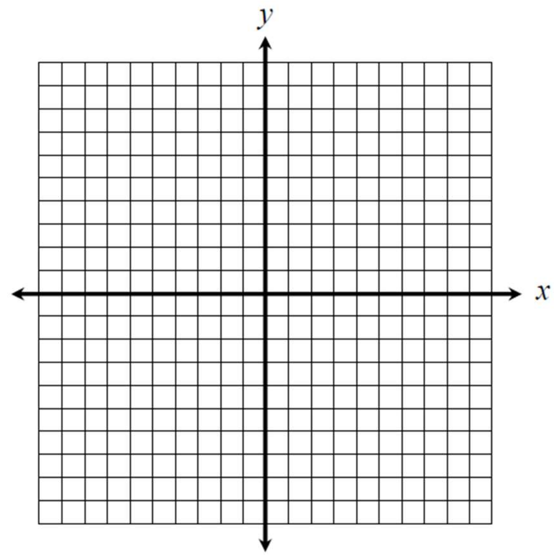
*Which of the functions in example 2 have holes? How do you know?

 Name ___________________________

Graphing Rational Equations Practice 2

Identify the key characteristics of the function and graph it.

1. $f(x) = \frac{x-1}{2x-4}$



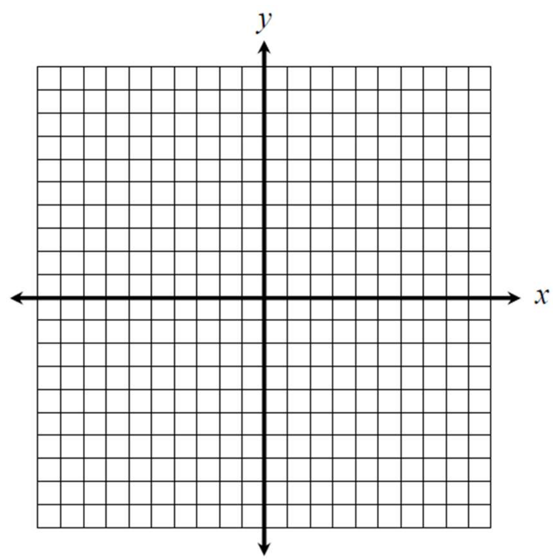
Hole(s):

VA: x-int:

HA: y-int:

SA:

2. $f(x) = \frac{x^3-9x}{x^2-4x}$



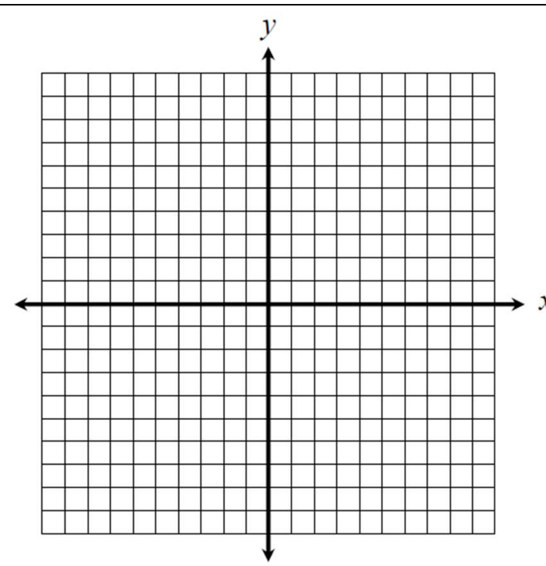
Hole(s):

VA: x-int:

HA: y-int:

SA:

3. $f(x) = \frac{4x-4}{x^2+x-6}$



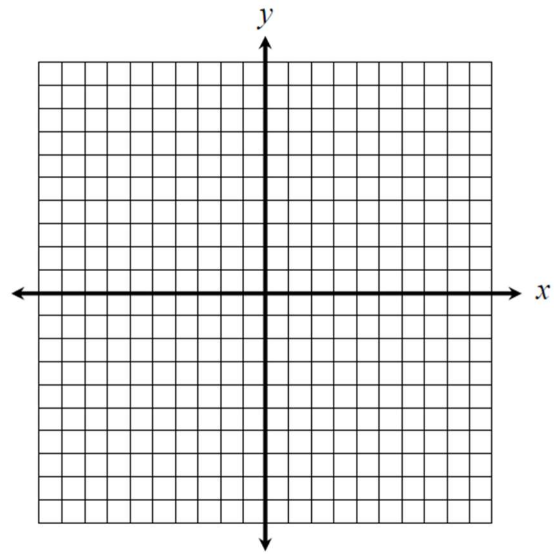
Hole(s):

VA: x-int:

HA: y-int:

SA:

4. $f(x) = \frac{x^2-4x+3}{x-4}$



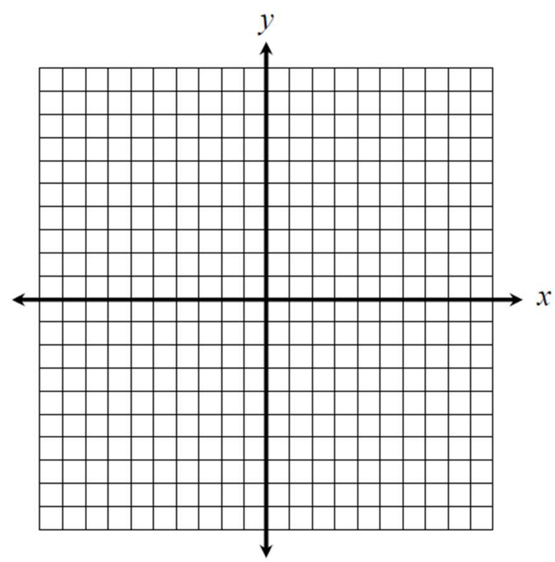
Hole(s):

VA: x-int:

HA: y-int:

SA:

5. $f(x) = \frac{x^2-7x+12}{x^2-5x+6}$



Hole(s):

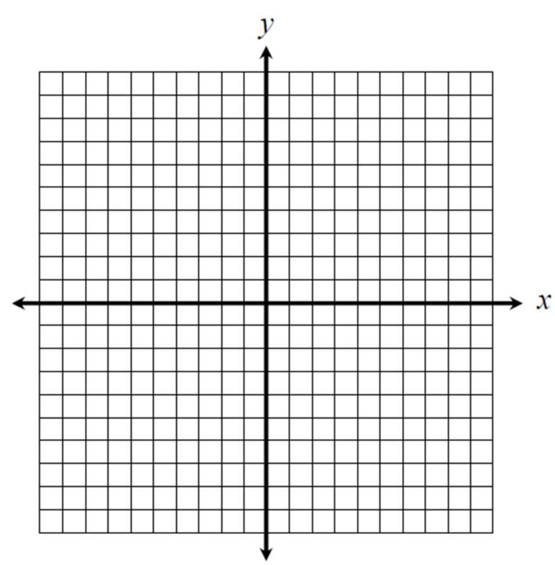
VA: x-int:

HA: y-int:

SA:

Domain: Range:

6. $f(x) = \frac{x^2-2x}{x-1}$



Hole(s):

VA: x-int:

HA: y-int:

SA:

RATIONAL *Functions*

EQUATION FORM:

X-INTERCEPTS:

VERTICAL ASYMPTOTES:

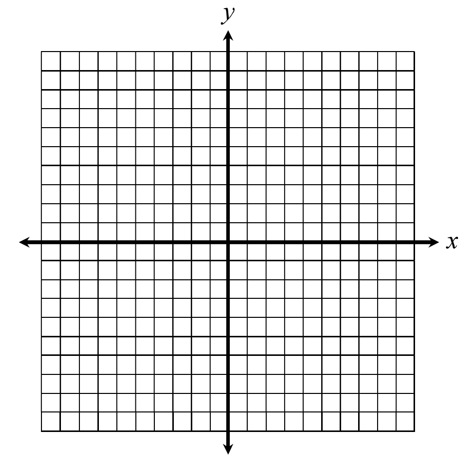
HORIZONTAL ASYMPTOTES:

- If degree of p > degree of q:
- If degree of p < degree of q:
- If degree of p = degree of q:

HOLES:

EXAMPLES

1 $f(x)=\dfrac{x^2-9}{x+2}$

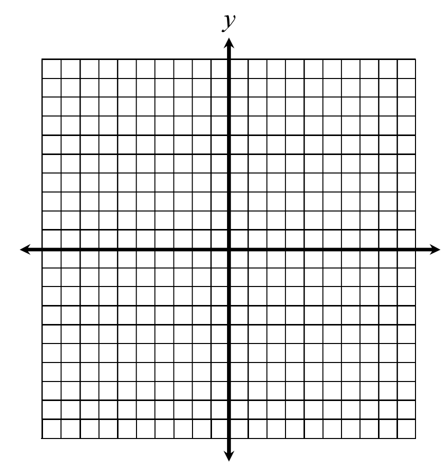


x-int: ______________________ **Holes:** ______________________

Vertical Asymptote: ____________ **D:** ____________________

Horizontal Asymptote: ___________ **R:** ____________________

2 $f(x)=\dfrac{x^2-4x-5}{x^2-1}$



x-int: ______________________ **Holes:** ______________________

Vertical Asymptote: ____________ **D:** ____________________

Horizontal Asymptote: ___________ **R:** ____________________

Name: ______________________________

Date: ______________________ Bell: ______

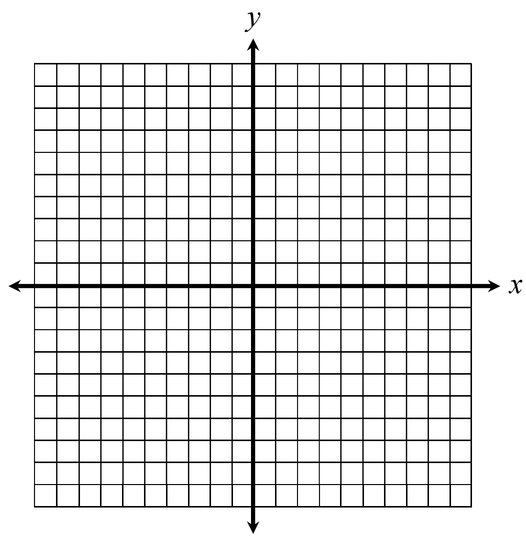
Algebra 2 Unit 4B

Graphing Rational Equations Practice

**** This is a 2-page document! ****

Graph each function. Identify the domain, range, asymptotes, and holes.

1. $f(x) = \dfrac{x+3}{x-2}$



***x*-int:** ____________________

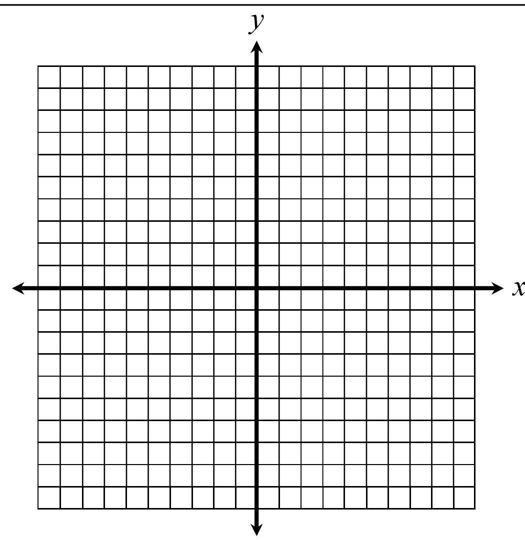
VA: ____________________

HA: ____________________

Hole: ____________________

y-int: ____________________

2. $f(x) = \dfrac{4x-24}{2x-4}$



***x*-int:** ____________________

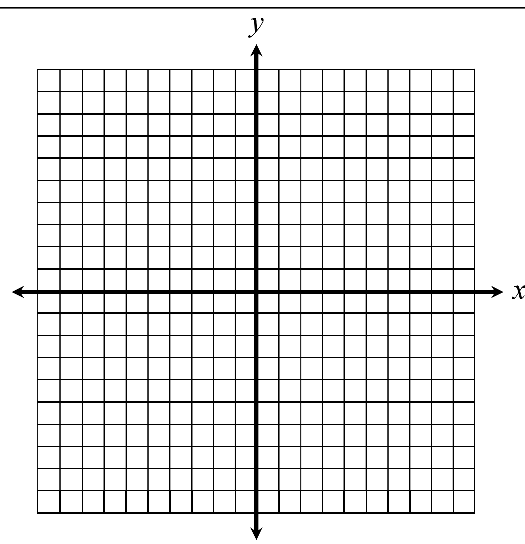
VA: ____________________

HA: ____________________

Hole: ____________________

y-int: ____________________

3. $f(x) = \dfrac{x^2+7x+10}{x+5}$



***x*-int:** ____________________

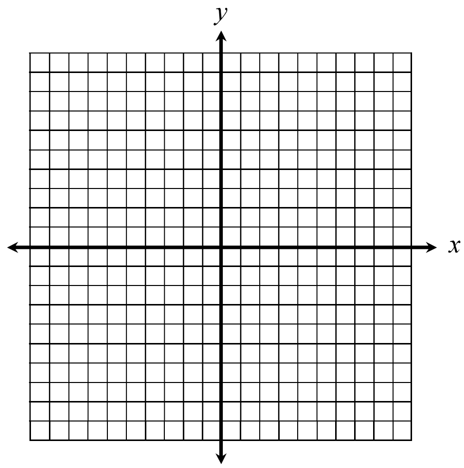
VA: ____________________

HA: ____________________

Hole: ____________________

y-int: ____________________

4. $f(x) = \frac{x+3}{x^2-9}$



***x*-int:** ______________________

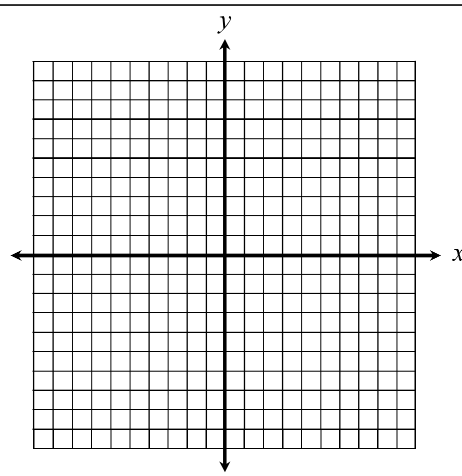
VA: ______________________

HA: ______________________

Hole: ______________________

y-int: ______________________

5. $f(x) = \frac{x^2+4x}{x^2+3x-4}$



***x*-int:** ______________________

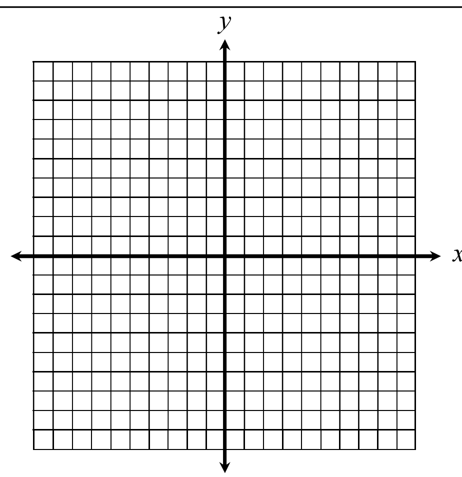
VA: ______________________

HA: ______________________

Hole: ______________________

y-int: ______________________

6. $f(x) = \frac{x^2-x-6}{x}$



***x*-int:** ______________________

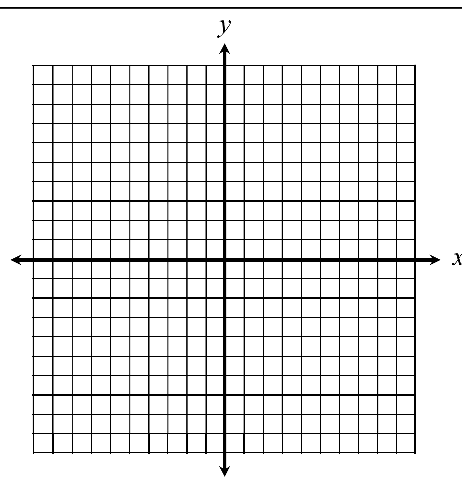
VA: ______________________

HA: ______________________

Hole: ______________________

y-int: ______________________

7. $f(x) = \frac{3x^2-5x-2}{x^2-3x+2}$



***x*-int:** ______________________

VA: ______________________

HA: ______________________

Hole: ______________________

y-int: ______________________

Determine whether the statement is TRUE or FALSE. If it is FALSE, choose the correct word to place in the blank to make it TRUE.	
Word Bank	
x y coefficient factor zero one two less than greater than	
1. To find the y-intercept of a function, you must substitute zero for y.	TRUE / FALSE ____________________
2. The reason there are vertical asymptotes in the graph of a rational function is because the denominator of a fraction may not equal zero.	TRUE / FALSE ____________________
3. To determine whether a rational function has holes, you must factor both the numerator and denominator, to see if there are any common factors.	TRUE / FALSE ____________________
4. The equation of a horizontal line contains only one variable, and that variable is x.	TRUE / FALSE ____________________
5. A slant asymptote occurs when the degree of the numerator is less than the degree of the denominator by 1.	TRUE / FALSE ____________________

Multiple Choice: Write the letter of the correct response in the space provided.

______ 6. Which describes the horizontal asymptote of the function $f(x) = \frac{x+4}{4x^2-5}$?

A. $y = 0$ B. $y = 1$ C. $y = 2$ D. $y = \frac{1}{2}$

______ 7. Which describes the slant asymptote of the function $f(x) = \frac{x^2+7x+1}{x-1}$?

A. $y = x + 3$ B. $y = x + 8$ C. $y = x + 6$ D. $y = x + 1$

______ 8. Which describes one of the vertical asymptotes of the function $f(x) = \frac{6}{x^2-9}$?

A. $x = 0$ B. $x = 1$ C. $x = 2$ D. $x = 3$

9. Given the function $f(x) = \frac{x^2+4x+3}{x+1}$, what is the y-coordinate of the hole?

10. What values of x are excluded from the domain of $f(x) = \frac{x^2-5x-6}{x^2-1}$?

11. Given the function $f(x) = \frac{x^2-5x-6}{x^2-1}$...

a. what is the x-intercept of the function?

b. what is the y-intercept of the function?

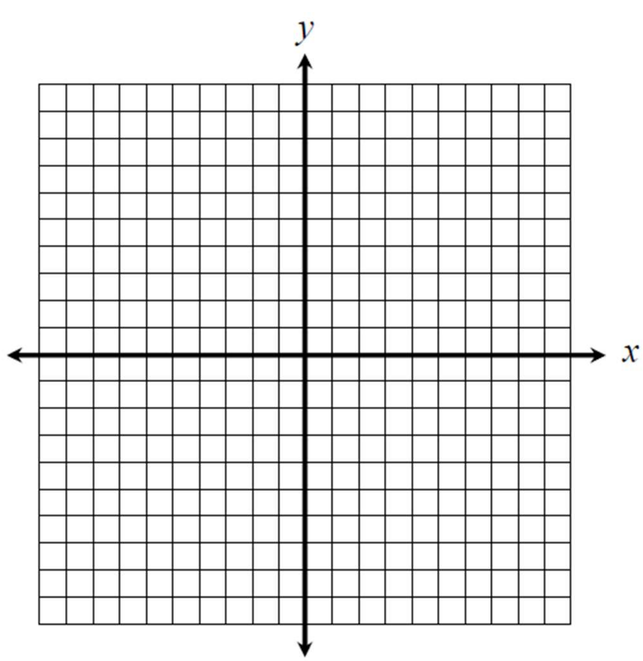
12. Matching: Match the function and the correct asymptote.

Function
$f(x) = \dfrac{2x-1}{x}$
$f(x) = \dfrac{x}{2x^2-2}$
$f(x) = \dfrac{3x}{x-2}$
$f(x) = \dfrac{x+1}{2x-1}$
$f(x) = \dfrac{x^2-4x+3}{x+2}$

Asymptote
$y = 0$
$y = \dfrac{1}{2}$
$y = x + 2$
$x = 2$
$x = 0$

13. Identify the characteristics of the function and graph it.

$$f(x) = \frac{x^2-2x-15}{x^2-9}$$



Hole(s):

VA:

x-int:

HA:

y-int:

SA: