MATH 1342 – STATISTICS
SECTION 8.1: A SINGLE POPULATION MEAN USING THE NORMAL DISTRIBUTION

- Using sample data to make generalizations about an unknown population is called **inferential statistics**.
- A **confidence interval** is another type of estimate but, instead of being just one number, it is an interval of numbers. It provides a range of reasonable values in which we expect the population parameter to fall. There is no guarantee that a given confidence interval does capture the parameter, but there is a predictable probability of success.
- To construct a confidence interval for a single unknown population mean μ, where the population standard deviation is known, we need $\bar{x}$ as an estimate for μ and we need the margin of error. Here, the margin of error (EBM) is called the *error bound for a population mean* (abbreviated EBM). May see ME
- Recall: $\bar{x}$ and s are called statistics. The sample mean $\bar{x}$ is the **point estimate** of the unknown population mean μ. Likewise, the sample standard deviation s is the **point estimate** of the unknown population standard deviation σ.
- NOTE: We can also consider margin of error for a proportion. We'll refer to this as error bound of the proportion, EBP. The sample proportion, $\hat{p}$, is the point estimate of the unknown population proportion p.
- The confidence interval estimate will have the form:

  (point estimate - error bound, point estimate + error bound) = $(\bar{x}-EBM, \bar{x}+EBM)$ or $(\hat{p}-EBP, \hat{p}+EBP)$
- The margin of error $(EBM$ or $EBP)$ depends on the confidence level (abbreviated CL). The **confidence level** is the percent of confidence intervals that contain the true population parameter when repeated samples are taken.
  Most often, it is the choice of the person constructing the confidence interval to choose a confidence level of 90% or higher because that person wants to be reasonably certain of conclusions drawn from the data.
- There is another probability called alpha (α). α is the probability that the interval does not contain the unknown population parameter.
  Mathematically, $\alpha + CL = 1$.

Ex. 1: Suppose we have data from a sample. The sample mean is 15, and the error bound for the mean is 3.2. What is the confidence interval estimate for the population mean?

Ex. 2: The average number of chips per bag from the potato chips available at a supermarket is unknown. A random sample of potato chip bags from the supermarket yields a sample mean of $\bar{x} = 250.6$ chips. Assume the sampling distribution of the mean has a standard deviation of $\sigma_{\bar{x}} = 18.9$ chips. Use the Empirical Rule to construct a 99.7% confidence interval for the true population mean number of potato chips.

Ex. 3: Suppose we know that a confidence interval for a population proportion is $(0.22, 0.40)$.What is the sample proportion, $\hat{p}$? What is the margin of error?

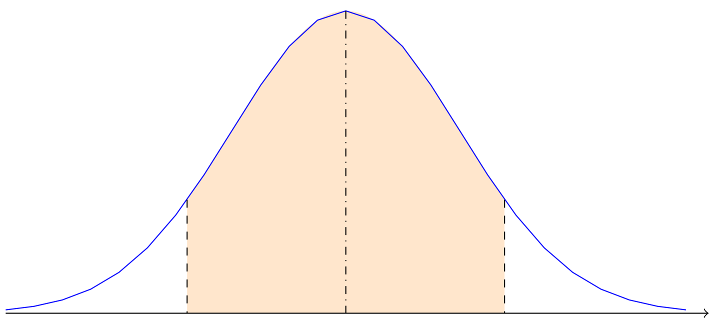
- Must use appropriate "standard deviation" for the parameter we are estimating. When working with sample means, need $\frac{\sigma}{\sqrt{n}} \Longrightarrow$ "standard error of the mean"
- Calculating the Confidence Interval: To construct a confidence interval estimate for an unknown population mean, we need data from a random sample. Steps:

(1.) Calculate the sample mean $\bar{x}$.

(2.) Find the z-score for the Stated Confidence Level: When know the population standard deviation σ: use a standard normal distribution to calculate the error bound EBM and construct the confidence interval.
Find z-value that puts an area equal to the confidence level (in decimal form) in the middle of the standard normal distribution $Z \sim N(0,1)$.
The confidence level, CL, = area in the middle of the standard normal distribution. $CL = 1 - \alpha$, so α is the area that is split equally between the two tails. Area of each tail $= \frac{\alpha}{2}$.

(3.) Calculate the Error Bound (EBM): The error bound for unknown population mean μ when the population standard deviation σ is known: $EBM = \left(z_{\alpha/2}\right)\left(\frac{\sigma}{\sqrt{n}}\right)$

(4.) Construct the Confidence Interval: The confidence interval estimate has the format: $(\bar{x} - EBM, \bar{x} + EBM)$.
Graphically, this looks like the following:



(5.) Write the Interpretation: The interpretation should clearly state the confidence level (CL), explain what population parameter is being estimated (here, a population mean), and state the confidence interval (both endpoints).
"We estimate with _______% confidence that the true population mean (include the context of the problem) is between _______ and _______ (include appropriate units).

Ex. 4: Find the z-score that relates to a 95% confidence level.

Common desired Confidence Intervals with their z-scores:
$CL = 90\%$: $z_{0.05} = 1.645$ $CL = 95\%$: $z_{0.025} = 1.96$ $CL = 99\%$: $z_{0.005} = 2.576$

To Find a Confidence Interval for a Population Mean for a Normal Distribution	
Given parameters:	Given data:
[STAT] $\rightarrow\rightarrow$ [TESTS] $\downarrow\downarrow\downarrow\downarrow$ 7:ZInterval.	[STAT] $\rightarrow\rightarrow$ [TESTS] $\downarrow\downarrow\downarrow\downarrow$ 7:ZInterval.
Inpt: $\rightarrow$ Stats	Inpt: $\rightarrow$ Data
Enter σ, $\bar{x}$, n, and C-Level (as a decimal).	Enter σ, List: (usually L1), Freq: (usually 1), and C-Level (as a decimal).
$\downarrow$ Calculate [ENTER].	$\downarrow$ Calculate [ENTER].
The confidence interval is displayed.	The confidence interval is displayed.
Recall this is $(\bar{x}-EBM, \bar{x}+EBM)$.	Recall this is $(\bar{x}-EBM, \bar{x}+EBM)$.

Ex. 5: Suppose average pizza delivery times are normally distributed with an unknown population mean and a population standard deviation of six minutes. A random sample of 28 pizza delivery restaurants is taken and has a sample mean delivery time of 36 minutes.

Find an 80% confidence interval estimate for the population mean delivery time.
$\sigma = 6$
$\bar{x} = 36$
$n = 28$
CL$=$ .80

SOLUTION:
Confidence Interval for 80% CL: $(34.547, 37.453)$

Find an 90% confidence interval estimate for the population mean delivery time.
$\sigma = 6$
$\bar{x} = 36$
$n = 28$
CL$=$ .90

SOLUTION:
Confidence Interval for 90% CL: $(34.135, 37.865)$.

We are 80% confident that the population mean of pizza delivery times is between ________________ and ________________ minutes when the sample size is 28 restaurants.
We are 90% confident that the population mean of pizza delivery times is between ________________ and ________________ minutes when the sample size is 28 restaurants.

⋆ NOTE: Compare these examples. How did the confidence interval change when we increased the confidence level?

Ex. 6: Refer back to the pizza-delivery problem. The mean delivery time is 36 minutes and the population standard deviation is six minutes. Assume the sample size is changed to 50 restaurants with the same sample mean. Find a 90% confidence interval estimate for the population mean delivery time.
$\sigma = 6$
$\bar{x} = 36$
$n = 50$
CL$=$ .90

SOLUTION: $(34.604, 37.396)$
We are 90% confident that the population mean of pizza delivery times is between ________________ and ________________ minutes when the sample size is 50 restaurants.

⋆ NOTE: Compare these examples. How did the confidence interval change for a confidence level of 90% when we increased the sample size?

- Calculating the Sample Size n: Can solve error bound formula to calculate the required sample size to give a specific margin of error: $n = \dfrac{z^2\sigma^2}{EBM^2}$ Round up to nearest integer

- In this formula, z is $z_{\alpha/2}$, corresponding to the desired confidence level.
  We use `invNorm` to find the z-score for the appropriate $\alpha/2$

Ex. 7: Refer back to the pizza-delivery problem. The mean delivery time is 36 minutes and the population standard deviation is six minutes. If we want to be 95% confident that the sample mean time is within 2 minutes of the true population mean time, how many randomly selected pizza delivery restaurants must be surveyed?

Ex. 8: The table shows a different random sampling of 20 cell phone models and their Specific Absorption Rate (SAR), which is the amount of radio frequency energy absorbed by a user's body. Use this data to calculate a 93% confidence interval for the true mean SAR for cell phones certified for use in the United States. Assume that the population standard deviation is $\sigma = 0.337$.

Phone Model	SAR	Phone Model	SAR
Blackberry Pearl 8120	1.48	Nokia E71x	1.53
HTC Evo Design 4G	0.8	Nokia N75	0.68
HTC Freestyle	1.15	Nokia N79	1.4
LG Ally	1.36	Sagem Puma	1.24
LG Fathom	0.77	Samsung Fascinate	0.57
LG Optimus Vu	0.462	Samsung Infuse 4G	0.2
Motorola Cliq XT	1.36	Samsung Nexus S	0.51
Motorola Droid Pro	1.39	Samsung Replenish	0.3
Motorola Droid Razr M	1.3	Sony W518a Walkman	0.73
Nokia 7705 Twist	0.7	ZTE C79	0.869

SOLUTION: $(0.80351, 1.0766)$